La Trasposizione Meta-Didattica: evoluzione di un quadro teorico per l'analisi dei processi di collaborazione tra insegnanti di matematica e ricercatori

The Meta-Didactical Transposition: evolution of a theoretical frame to analyse collaborative processes between mathematics teachers and researchers

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INTRODUCTION

Education is a "total fact" in the sense given by the anthropologist Marcel Mauss (1923/2007) that is to say "a fact that set in motion the totality of society and its institutions" (p. 102). It is therefore a complex phenomenon that needs to be studied in its entirety, and in particular by considering all those involved in education as voices that need to be taken into account. This epistemological concern leads us to consider teachers, teacher educators, researchers as key actors in educational research. To understand the complexity of the phenomena involved, it is essential to consider the points of view of all these actors, when they are together involved, as part of different institutions, in professional development programs or research projects. The need to investigate such complexity has been particularly emerging in the current century as a field of research (Adler et al., 2005; Sfard, 2005), with a specific focus on the investigation of the processes that characterise the ways in which teachers learn when they are involved in a collaborative work (Robutti et al., 2016; Robutti et al., 2024), where collaboration is used in its etymological meaning by Latin (*cum laborare*).

This investigation requires to study the interactions between all the actors involved in such phenomena in order, on the one hand, to understand and to enhance the mechanics of professional development, and, on the other hand, to enable every actor to become aware of each other's contributions to academic research. With this goal, we developed a theoretical framework aimed at supporting us in the investigation of these complex processes.

The working hypotheses behind the development of this framework were based on three fundamental ideas. First of all, the framework should account for the complexity of the phenomena involved by considering different theoretical perspectives to interpret these phenomena. Second, the framework should also account that all the actors involved in education have an equal voice to participate in collective reflection since the institutions to which they belong have an equal importance in the educational processes. Third, as these voices are echoes of the institutions to which the actors belong, the framework should take into account the institutional affiliations of the actors, as well as their professional aims.

Bearing in mind that the field of our research concerns mathematics education, we have seen that the framework of the Anthropological Theory of Didactics (Chevallard, 1985) aligns with these fundamental ideas. For this reason, we referred to it with the aim of expanding it to encompass not only didactic, but also meta-didactical considerations, in the sense of *reflecting on* the didactic dimension of the teaching and learning mathematics at school.

In the years 2008-2012, a team of scholars from Turin and Modena and Reggio Emilia Universities (Arzarello et al., 2014) felt the need to share ideas and theoretical constructs to study the complexity and the richness of the Italian research for innovation (Arzarello & Bartolini Bussi, 1998), for which the collaboration between mathematics teachers and researchers plays a key role. A research framework useful to investigate this collaboration was presented by this team at the National Seminar in Didactics of Mathematics of the association AIRDM (http://www.seminariodidama.unito.it/mat12.php), and then in an international publication (Arzarello et al., 2014).

The framework, named Meta-Didactical Transposition (MDT.1), was aimed at investigating over time the processes of teachers and researchers' interaction, focusing both on the practices developed by mathematics teachers and researchers when they collaborate in various ways and in different contexts and the ways in which they discuss on these practices and justify the choices made when these practices are developed.

THE MDT.1 FRAMEWORK

The ideas guiding the development of MDT.1 were:

a) Considering the importance of the institutions in which mathematics teachers collaborate with researchers, by referring to the Anthropological Theory of Didactics (Chevallard, 1985).
b) Focusing on the work of teachers in communities of inquiry (Jaworski, 2008) to take into account the organisation of the main professional development programs carried out in Italy (e.g., m@t.abel, Piano Lauree Scientifiche, MOOCs, …), in which teachers collaborate with each other in small/large communities.
c) Focusing on the interaction between communities of teachers and communities of researchers, involved in professional development programs or action-research projects/groups.
d) Taking into account the reciprocal influences between the communities of teachers and the communities of researchers, in line with our research context (Arzarello & Bartolini Bussi, 1998).
e) Investigating the evolution of the practices developed when the two communities interact in a dynamic way, looking at the processes accompanying their evolution and at the role played by their reciprocal influences.

MDT.1 is based on the Anthropological Theory of Didactics, which considers that, in the final analysis, all human activity in institutions consists of praxeologies. A praxeology is made of four components: a type of task, a technique to accomplish the task, a technology (the word technology being considered in its primary meaning: technología (τεχνολογία) téchnē (τέχνη), 'craft' and -logos (λόγος), 'language', ability to communicate, discourse about the technique) that justifies the technique, and a theory on which the technology is based.

In the context of mathematics education, this praxeology could be:

- mathematical, when it aims to characterise the human activity around the accomplishment of a mathematical task.
- didactical, when it aims to characterise the human activity around the accomplishment of a didactical task.

A didactical praxeology models the practices and knowledge involved in teaching to bring out a mathematical praxeology in the classroom.

For knowledge to be taught at school, it has to be selected, arranged and restructured to suit the conditions of learning. The term *didactical transposition* is used to refer to the process of transformation from 'scholarly' knowledge to knowledge 'to be taught'.

In MDT.1 the term Meta-Didactical Transposition refers to the transposition of 'scholarly' knowledge developed by research in mathematics education to knowledge to be included in didactical praxeologies.

The MDT.1 framework is characterised by five main components. The first component, in line with the aim behind the development of MDT.1, is the role played by institutional aspects in the processes of interaction between communities of teachers and researchers, as both communities are immersed in an institution, which requires the members of the communities to behave according to specific conditions and constraints. The word 'institution' has to be taken in the Chevallard's meaning: "Behind the persons, and the knowledge, there appeared the institutions, to be regarded on a par with the persons, in the light of a dialectic between persons and institutions. Persons are the makers of institutions which in turn are the makers of persons" (Chevallard, 2007, p. 132).

The second component is the notion of meta-didactical praxeologies, which refers to meta-didactical tasks introduced within teacher education programs and action-research projects and to techniques, technologies and theories that refer to the knowledge about the didactic system. "Using MDT, the activities and knowledge of teachers and didacticians [researchers also acting as teacher educators, see Jaworski and Potari (2021)] are also modelled with the notion of praxeology: they are called *meta-didactical praxeologies* as they refer to knowledge *about* the didactic system. *Teachers praxeologies* and *didacticians praxeologies* are different instances of meta-didactical praxeologies" (Minisola et al., 2024, p. 6). In fact, "Within meta-didactical praxeologies, what is under scrutiny is not the didactics in the classroom, but the practices and the theoretical reflections developed in teacher education activities" (Aldon et al., 2013, p. 101).

The third element is constituted by the dynamics between external and internal components. Meta-Didactical Transposition introduces a dynamic model which relies heavily on the fact that certain components of the praxeologies of the two communities change status over time. These components could change their status, from external to internal status, according to the community under study. The process that allows teachers and researchers to introduce external components into their own praxeologies is called internalisation.

The fourth component is the notion of double dialectic. It considers the fact that the members of the communities of teachers and researchers interacting together have the opportunity to experiment two dialectics. The first dialectic arises at a didactical level, between the personal meaning that students attribute to the teaching situation and its shared scientific meaning. The second dialectic is at a meta-didactical level, between the interpretation given to the didactical dialectic by the various actors involved in the Meta-Didactical Transposition process, based on their respective praxeologies.

Finally, the fifth element is the role of brokers played by specific members of the interacting communities. The driving force behind the dynamics that characterise the Meta-Didactical Transposition processes is, in fact, the opportunity given to the actors to link their respective praxeologies. This opportunity is the result of the actions of specific members of the communities, who act as brokers, fostering a dialogue and the sharing of ideas: "Brokers facilitate the sharing of knowledge and practices from one community to the other […]" (Aldon et al., 2013, p. 104).

One crucial idea within MDT.1 is that, thanks to the internalisation process, the praxeologies of teachers and researchers could hopefully evolve towards new praxeologies that share common elements. This evolution leads to what we call *shared praxeologies*, which foster a change in teachers and researchers' positioning within their institutions. For this reason, it is important to investigate both the internalisation process and its effects in terms of new and shared praxeologies.

The evolution of praxeologies involves changes at both the praxis (tasks and techniques) and the logos (technologies and theories) levels, reflecting each community's professional knowledge and experience.

As stated above, initially applied to analyse professional development processes, MDT.1 was soon extended to study collaborative research contexts, broadening its application to include members from various fields, such as computer science and design (Cusi et al., 2022). This broader application spurred further reflections on how MDT.1 could be adapted to explore new environments, prompting the integration of additional theoretical elements.

In particular, to better develop the investigation of the internalisation processes realised within these different contexts, MDT.1 needed to be integrated with other important elements that could deepen the study of the process of internalisation from different viewpoints:

1. The lens of *connectivism* to describe internalisation as a process that supports the construction and expansions of the actors' networks of knowledge;
2. The notion of *agents*, at the micro-level, to identify the driving forces that support the evolution of specific praxeological components at the macro-level;
3. The notion of *boundary object* and the framework that characterises the actions that could be carried out on boundary objects, to investigate how internalisation processes are boosted by teachers and researchers' joint actions on a common object (methodological, epistemological, theoretical…).

The new framework, integrated with these elements, is called MDT.2 (Cusi et al., 2022).

MDT.2 TO DEEPEN THE INVESTIGATION OF THE INTERNALISATION PROCESS

In the following, we detail the integrations of new theoretical elements within MDT.1 that led to MDT.2. The presentation of these new elements will be done by introducing different theoretical perspectives for examining interactions among diverse communities of teachers and researchers in various collaborative settings. These perspectives are aimed at enhancing the analysis of the internalisation process within MDT.1, specifying *where*, *why* and *how* it happens (Cusi et al., 2022). This theoretical integration supports a richer interpretation of how learning could happen within professional development or collaborative research environments, highlighting key aspects of the interaction between community members. In this way, MDT.1's evolution has led to a framework, MDT.2, that aims at being more flexible in describing, analysing and interpreting the internalisation processes that are realised within collaborative contexts.

The WHERE of internalisation

The study on the WHERE of internalisation has started from the application of MDT.1 to an online teacher professional development context, specifically the Math MOOC UniTo project (2015-2020) by the University of Turin (Taranto & Arzarello, 2020; Taranto et al., 2020). Here, two communities—educators and a large group of teachers from across Italy—engaged without direct orchestration, allowing teachers to share ideas and resources freely. This unstructured interaction enriched the MOOC, fostering teachers' internalisation of praxeological elements. Given the unique environment of MOOCs, studying the internalisation process within MDT.1 required integrating connectivism theory (Siemens, 2005) to interpret the complex interactions. Connectivism frames learning as the evolution of a knowledge network, where participants self-organise, integrate new resources, and share within the group. In MOOCs, teachers expand their network of knowledge by adding new "nodes" (e.g., information, ideas) and forming new connections, fostering ongoing learning. More generally, within MDT.2, this evolution reflects two processes that can be realised also in other contexts: teachers internalising new elements into their meta-didactical praxeologies, and reinterpreting existing ones, which can lead to changes in their didactical praxeologies (e.g., Taranto et al., 2020). Interpreting learning as the evolution of the individual's own network of knowledge enables, in fact, to conceive internalisation as a phenomenon that happens within a wide context, which includes not only the institutions to which the individual belongs, but also the complex networks of their interactions.

The WHY of internalisation

The need to explore the WHY of internalisation arose in the context of a professional development course which promoted a specific didactical use of GeoGebra. In the end, as it might happen in any professional development context, the course had very different outcomes on the praxeologies of the participating teachers (Prodromou et al., 2018). Some of them integrated the promoted use of GeoGebra as a component of their didactic praxeologies; other teachers experimented with the proposed activities but decided not to adopt them in their daily praxeologies. It arose therefore the need to highlight the causes that determine a successful or unsuccessful internalisation process.

To this purpose, MDT.1 was integrated with a new theoretical element, which allowed us to study the professional development process and product as it happens for physical phenomena related to gas. In physics, two levels of variables are distinguished: macro variables (e.g., temperature, pressure or volume of the gas) and micro variables (e.g., mass or velocity of a particle). The interaction of micro variables is observable as the emergence of macro variables. Similar phenomena happen within professional development contexts, where interactions among several smaller and simpler agents at the micro-level can influence the internalisation process, at the macro-level (Goldstein, 1999). A micro-level of analysis was then added to MDT.1, where different agents may be active or activated to interact with each other. This interaction may (or not) determine teachers' praxeologies development and evolution, and this is visible as a product at the macro-level. With the term *agents* we refer to human or non-human entities involved in the mathematical activity (de Freitas & Sinclair, 2014). Among others, we identified methodological, institutional, material, technological, and motivational agents (Prodromou et al., 2018), but also other kinds of agents can be involved at the micro level.

When a teacher is planning and teaching, the interaction between different agents, at the micro-level, contributes to shaping the teacher's praxeologies or some of their components (techniques, technologies or theories), at the macro-level. The introduction of this micro-level of analysis could enable researchers to identify the driving forces that trigger or sometimes inhibit the evolution of specific praxeological components characterising the internalisation process.

The HOW of internalisation

To investigate the HOW of internalisation, the notion of boundary object (BO) and a framework to interpret the processes that are developed at the boundary were integrated within MDT.1. These new theoretical elements support the analysis of the discourses that the teachers and researchers develop when working on a common object and enable to deepen the interpretation of the evolution of specific praxeological components.

The concept of BO originated in an ethnographic study of the coordination mechanisms of scientific work (Star & Griesemer, 1989). It has three essential characteristics: interpretative flexibility, material structure or organisation, and granularity. The BO is understood as a device that enables joint work to be initiated between several communities and ensures sufficient flexibility for everyone to find an interest in its study or use. It should be noted that this interpretation, this arrangement, can only concern an object that has professional importance in each of the communities.

Including this notion within MDT.1 enabled us to conceive the boundaries of the BO as the collaborative contexts within which the people involved in the work on BO have opportunities to exchange and pursue common aims. In the context of MDT, BOs represent the focus of the interactions among different communities and the work on BO is considered a driving force that triggers the evolution of their praxeologies. This evolution happens when a property, a concept, an experience or a piece of knowledge contributed by one or other of the communities within the joint work on a BO is internalised by the members of the interacting communities. Internalisation is conceived as the result of the communities' discussions and reflections on their joint work on the BO, as these processes make the objects of reflection become internal components of the communities' didactical or meta-didactical praxeologies.

To describe HOW this evolution happens and to highlight evidence of the internalisation process within the discourses carried out within the interacting communities when they collaboratively work on a common BO, we referred to Carlile's (2004) characterisation of the types of actions that could be carried out on BOs and are likely to modify boundaries by expanding the space of shared understanding of the BO by the different communities.

The first action is knowledge *transfer* and relates to communications whose validity is not contested by the speakers and for which the boundary remains stable. In other words, transfer refers to interactions which may produce disagreements, but for which the exchanges concern concrete, syntactic aspects of the boundary object.

The second action is *translation* and is carried out at the semantic level, as the focus of communication among actors is a negotiation of the meaning of certain components of the BO on which the different actors are jointly working. The search for a common, shared meaning leads to changes in points of view, with the object acting as a cognitive mediator between the communities.

The third action is *transformation* and it is associated with social mediation, in which the interests of the communities pragmatically construct the boundary space that then becomes socially recognised. This action is carried out at a pragmatic level and is aimed at enabling the actors to integrate specific BO's components within their practices. The knowledge at stake is then transformed by negotiation between the actors to create a new shared knowledge: "When interests are in conflict, the knowledge developed in one domain generates negative consequences in another. Here the costs for any actor are not just the costs of learning about what is new, but also the costs of transforming 'current' knowledge being used (i.e., common and domain-specific knowledge)" (Carlile, 2004, p. 559).

Referring to Carlile's characterisation enable to show HOW the discourses between the interacting communities aimed at understanding the BO contribute to foster the internalisation process that modifies their praxeologies and leads to a "shared praxeology" characterised by those components that are understandable and familiar by both the actors' new praxeologies.

NEW ELEMENTS TO TAKE INTO ACCOUNT: A FURTHER EVOLUTION OF MDT.2

In the last few years, the MDT.2 framework has further evolved thanks to other studies that suggested taking into account other important aspects to be integrated, such as the role played by beliefs within the processes under investigation, the design of teaching materials that accompany the collaborative work, and the interplay between the different institutional roles played by the different actors.

A theoretical framework that appears to be compatible with MDT and that allows us to focus on specific aspects of meta-didactical praxeologies related to the design of teaching materials is the Documentational Approach to Didactics (DAD) (Gueudet & Trouche, 2009). The DAD framework was originally aimed at studying teachers' work and professional development, but it has also been applied to the case of teacher educators generating documents for their work with teachers during a professional development program. In particular, Pocalana and Robutti (2024a) proposed an analysis of the work of didacticians relying on a combination of the MDT and the DAD frameworks. In their interpretative model, the didacticians' generation of documents for a teacher professional development program is conceptualised as part of their meta-didactical praxeologies for the design and implementation of the program itself, including elements of both the praxis and the logos components. Pocalana and Robutti (2024a) also found a double dialectic between didacticians' personal beliefs and the logos of their meta-didactical praxeologies, which reciprocally influence each other, because individuals are never entirely determined by their institutional position. Regarding their documentation work, didacticians' personal beliefs about the needs of teachers participating in a PD program influence both the choice of resources to rely on and their utilisation schemes. Indeed, the didacticians' personal level is intertwined with their institutional position as part of a community with shared praxeologies.

Pocalana and Robutti (2024b) also found that the evolution of teachers' beliefs during a professional development program is both a motivational agent and a consequence of the evolution of their meta-didactical praxeologies, which, in turn, is intertwined with the evolution of didacticians' meta-didactical praxeologies. This is coherent with Swan's (2007) conceptualisation of the double direction relationship between teachers' practices and beliefs, but it goes further, also taking into account the *logos* component of their praxeologies and their relationship with didacticians' praxeologies.

The research on possible agents of internalisation brought forth the necessity of networking MDT.2 with other theories to describe the more complex phenomena, and showed the potentiality of including within MDT.2 further elements of ATD: the concept of institutional *positions* and the *levels of didactic co-determinacy*. In Minisola and colleagues (2024), the interplay of didacticians and teachers' institutional *positions* brings forth the evolution of their meta-didactical praxeologies. Both of them, in fact, may cover at least two different positions:

- Teachers are the ones who teach the students, and simultaneously they are learners of how to teach and how to prepare teaching.
- Didacticians are the ones who teach the teachers and simultaneously the ones who research on teachers' professional development.

For the teachers, this was already modelled, in MDT.1, as the *double dialectic* that is established between didactical and meta-didactical levels. For the didacticians, it has been found that the double dialectic can develop between *the meta-didactical level* and *the research level:* the meta-didactical level is developed during teachers' professional development, and the research level is developed when they are making sense of the data collected during teachers' professional development.

THE SEMINAR

The aim of the seminar is to present the structure of the MDT framework by introducing all its elements as a result of the theoretical evolution described above (in the following, we will use MDT to refer to the results of the evolution from MDT.1 to MDT.2 to the further developments presented in the previous paragraphs). Moreover, we will reflect on the flexibility of the framework as a theoretical tool to investigate, from different perspectives, the collaborative interactions between teachers and researchers within professional development programs and research projects and the effects of these interactions in terms of evolution of teachers and researchers' praxeologies.

For this reason, during the seminar we will focus on a unique set of data and present different levels of analysis of these data by referring to the various components of our framework to analyse the data from different perspectives, showing the richness brought by this multi-level analysis that the MDT framework enables to develop.

The data have been collected within the context of a professional development program for in-service teachers developed and implemented at the University of Turin by two of the authors (Cusi et al., 2020). Five in-service teachers participated in the program: four of them from scientific oriented upper secondary school (grades 9 and 10) and one teacher from lower secondary school (grades 6 to 8).

The focus of the analysis are, in particular, the activities of scenario design developed by the five teachers in collaboration with the researchers who implemented the program.

We define scenario design as a process of envisioning possible implementations of mathematical classroom activities in which the interventions of the students and the teachers are made explicit. In particular, this process consists not only in designing the tasks for students and the teaching methodology, but also in hypothesising possible students' answers to the tasks and excerpts of classroom discussion, containing teachers' interventions. The product of this process is an ordered set of scenes - herein called scenario - representing the foreseen development of the classroom activity (i.e., teachers' interventions aimed at supporting students' learning processes, highlighting and discussing their difficulties, activating students' reflections...).

The data at disposal are the video-recordings of the meetings between the five teachers and the researchers, the transcripts of these meetings, the different versions of the written scenarios designed by the teachers, the final interviews during which teachers reflected on their experience on scenario design and on its effects on their professional development.

Through the analysis of these data, we will investigate the effects of the teachers and researchers' joint work on scenarios in terms of evolution of teachers' praxeologies, considering different perspectives thanks to the multiple lenses provided by MDT.

To conclude the seminar, we will discuss possible new directions of investigation related to the theme of the collaborative joint work between the communities of actors involved in the educational processes.

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